Wavelet-based Image Denoising with Optimal Filter

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Abstract: Image denoising is basic work for image processing, analysis and computer vision. This paper proposes a novel algorithm based on wavelet threshold for image denoising, which is combined with the linear CLS (Constrained Least Squares) filtering and thresholding methods in the transform domain. We demonstrated through simulations with images contaminated by white Gaussian noise that our scheme exhibits better performance in both PSNR (Peak Signal-to-Noise Ratio) and visual effect.

Keywords: *Image Denoising, Noise Reduction Wavelet*

1. Introduction

Digital images have applications in daily life, such as digital cameras, HDTV (High Definition Television) and in areas of research and technology including GIS (Geographical Information System). Datasets collected by image sensors are generally contaminated by noise and noise can be introduced by transmission errors and compression. The problem of image denoising is to recover an image that is cleaner than its noisy observation. Thus, noise reduction is an important technology in image analysis and the first step to be taken before images are analyzed [1].

Although wavelets have efficient noise reduction ability, wavelets still have problems on a heavy noisy network. We investigate the problem of image denoising when the source image is corrupted by additive white Gaussian noise, which is a valid assumption for images obtained through transmitting, scanning or compression.

In this paper we propose a simple and efficient algorithm based on the wavelet threshold for image denoising, which combines the linear filter applied to the LL subband and the thresholding method applied to the LH, HL, HH subbands in the transform domain.

The remainder of this paper is organized as follows. In the next section, related work and theoretical background about wavelet thresholding is presented. We describe our noise reduction algorithm in Section 3. In Section 4 we present some experimental results of our proposed scheme, and finally conclude with the conclusion in Section 5.

2. Related Work

There are two basic approaches to image denoising, which are the spatial filtering method and transform domain filtering method [1]. A traditional way to remove noise from a noisy image is to employ the spatial filter, but this works well only if the underlying signal is smooth. To overcome the weakness of the spatial filtering, a wavelet based denoising scheme is introduced [2]. Wavelets give a superior performance in image denoising due to properties such as sparsity and multiresolution structure.

Simple denoising algorithms that used the wavelet transform consist of the three steps [3].

Step 1. Calculate the wavelet transform of the noisy signal;

Step 2. Modify the noisy wavelet coefficients according to a rule;

Step 3. Compute the inverse transform using the modified coefficients;

One of the most well-known rules for step 2 is soft thresholding analyzed by [4]. Due to its effectiveness and simplicity, it is frequently used in the literature. The main idea is to subtract the threshold value T from all coefficients larger than T and to set all other coefficients to zero [3]. Generally, these methods used a threshold value that must be estimated correctly to obtain good performance.

And some researchers have considered wavelet shrinkage [2,5]. The basic idea is to model wavelet transform coefficients with prior probability distributions. Then, the problem can be expressed as the estimation of clean coefficients using *a priori* information with Bayesian estimation techniques, such as the maximum *a posterior* (*MAP*) estimator [6]. However, it has weak model for wavelet coefficients of natural images because they ignore the dependencies between coefficients, and its major problem lies in the difficulties in determining a proper shrinkage function and threshold [3,7].

2.1 Wavelet Filterbank Theory

Recently, filterbanks have found wider applications in image processing, such as coding and denoising [3]. In practice, the procedures of image decomposing based on filterbanks are similar to the wavelets image decomposition.

Let $a_k[n]$ be scaling coefficients of scale k and position n, and let $h[n]$ be the filter coefficients corresponding to the

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scale function. From wavelet theory [4], we know that

$$a_{k+1}[n] = \sum_{m} a_k[m] \cdot h[m-2n] \qquad (1)$$

where the coefficients $a_{k+1}[n]$ represent a coarser resolution then $a_k[n]$. Equation (1) indicates that the scaling function coefficients a_{k+1} scale can be obtained by convolving a reversed $h[n]$ with a_k and downsampling by two.

Similarly

$$d_{k+1}[n] = \sum_{m} a_k[m] \cdot g[m-2n] \qquad (2)$$

where $d_k[n]$ is the wavelet coefficients of scale k and position n. $g[n]$ is the set of filter coefficients corresponding to the wavelet. From equation (1) and (2), we can obtain increasingly coarser scales of scaling coefficients, a_{k+1}, and wavelet coefficients, $d_{k+1}[n]$, by convolving the scaling function coefficients, $a_k[n]$ by both a reversed scaling function filter, $h[n]$, a reversed wavelet filter, $g[n]$, and downsampling by two.

The reconstruction of finer scaling coefficients is obtained by

$$a_k[n] = \sum_{m} a_{k+1}[m] \cdot h[n-2m] + \sum_{m} d_{k+1}[m] \cdot g[n-2m] \qquad (3)$$

From equation (3), we can obtain an arbitrarily fine scale representation of a signal by upsampling the scale and wavelet coefficients, and filtering the coefficients with their respective filter, $h[n]$ and $g[n]$. Fig. 1 shows the general structure of decomposition and reconstruction. The $H(z^{-1})$ and $L(z^{-1})$ filters that are associated with decomposition form what is known as the *analysis filterbank* and $H(z)$ and $L(z)$ filters that effect reconstruction form the *synthesis filterbank* [5].

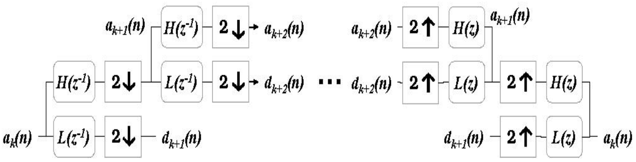


Fig. 1. Analysis and Synthesis Filterbanks

3. Proposed Algorithm, for Denoising

This section describes the method for computing the various parameters used to compute the threshold and our image denoising algorithm. The wavelet transform approach is used for the recovery of the corrupted image with optimal filter.

The threshold value (T_N), which is adaptive to different subband characteristics, is used to calculate the parameters as equation (4).

$$T_N = \frac{\beta \hat{\sigma}_n^2}{\hat{\sigma}_y} \qquad (4)$$

where scale parameter β is calculated once for each scale using $\beta = \sqrt{\log(L_k/J)}$ where L_k is the length of the subband at the k_{th} scale, J is the total number of decompositions and $\hat{\sigma}_y$ is the standard deviation of the subband. Noise variance $\hat{\sigma}_n^2$ is estimated, as in [8], using the robust median estimator of the subband of a Daubechies.

$$\hat{\sigma}_n^2 = \frac{median(|Y_i|)}{0.6745}, \quad Y_i \in \textit{each subband} \qquad (5)$$

where 0.6745 is the experiential value [9].

We have actually applied the filter to LL subband as a Constrained Least Squares (CLS) filter, instead of a minimum mean squares error (wiener) filter, because this filter is optimal for a general image while the wiener filter is optimal in an average scene [10]. This means that the wiener filter requires knowledge of the mean and variance of the noise, so we have to know the value of power spectrum of the noise, and a power spectrum of the undegraded image [11].

Here, a simple to implement and computationally more efficient algorithm is described. Starting with a noisy image, our completed denoising algorithm can be summarized as follows:

1. Decompose the image into subbands;
2. Estimate the noise variance in the noisy image using equation (5);
3. For each level, calculate the scale parameter β;
4. For each subband
   (a) Compute the standard deviation and threshold T_N using equation (4);
   (b) Apply soft thresholding to the subbands including LH_i, HL_i, HH_i;
   (c) Apply a CLS filter to the LL_i subband for removal noise coefficients;
5. Reconstruct the image from the denoised subbands;

4. Results

Performance of the noise reduction algorithm is measured using quantitative performance measures such as Peak Signal-to-Noise Ratio (PSNR) and in terms of visual quality of the images. Many of the current techniques assume the noise model to be Gaussian [1], and in this paper we have tested our approach on the noisy image with the Gaussian noise model [11].

The PSNR is given by

$$S = 20\log_{10}\left(\frac{256}{\varepsilon}\right), \quad \varepsilon = \sqrt{\frac{1}{N^2}(S_k - D_k)^2} \qquad (6)$$

where S is the PSNR in dB, N^2 is the number of pixels, S_k

and D_k are the original image and the denoised one respectively.

For the results in this work, we have implemented our method in MATLAB 7.0, and test our approach on the 512×512 Lena, Barbara, Boat and Goldhill 8-bit grayscale images, which are widely used in the image processing literature. Noisy images are corrupted by white Gaussian noise with Matlab imnoise function, described in [11]. This algorithm was tested using different noise levels and compared with the VisuShrink, SureShrink, BayesShrink, Sendur's method [12], OracleShrink, Wiener Filter, and Kaur's method [13] called NormalShrink.

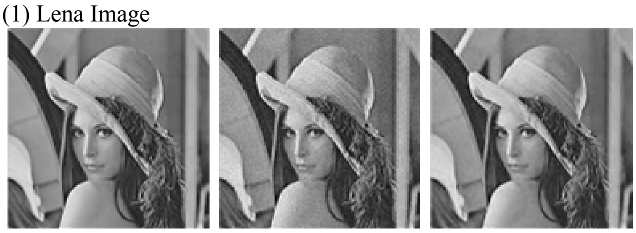
The PSNR results are shown in Table 1 and the numeric results on the column of our method are collected by average of five times.

Table 1. PSNR results [dB] for various test images and δ value

	δ	Visu-Shrink	Sure-Shrink	Bayes-Shrink	Sendur's Method	Oracle-Shrink	Wiener	Normal Shrink	Our Method
Lena	10	28.76	33.28	33.32	33.94	33.61	33.53	33.57	34.80
	20	26.46	30.22	30.17	30.73	30.38	30.35	28.98	31.53
	30	25.14	28.38	28.48	28.94	28.60	28.53	25.69	29.71
Barbara	10	24.81	30.21	30.86	31.13	31.50	31.37	29.81	32.54
	20	22.81	25.91	27.13	27.25	27.40	27.32	26.79	28.56
	30	22.00	24.33	25.16	25.21	25.32	25.22	24.29	26.48
Boat	10	26.49	31.19	31.80	32.25	N/A	N/A	N/A	32.41
	20	24.43	28.14	28.48	28.93	N/A	N/A	N/A	29.20
	30	23.33	26.52	26.60	27.11	N/A	N/A	N/A	27.36
Goldhill	10	N/A	31.87	31.90	N/A	31.97	31.71	31.80	32.31
	20	N/A	28.43	28.65	N/A	28.76	28.65	28.26	29.26
	30	N/A	27.02	27.11	N/A	27.16	27.09	27.09	27.69

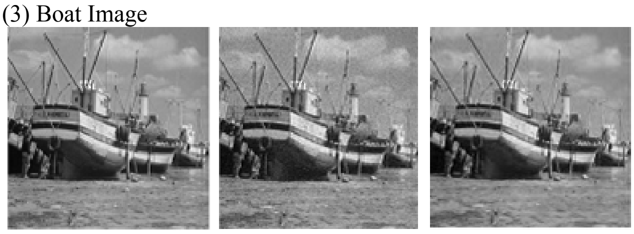
Fig. 2 shows the image results of each denoising method. Using our algorithm, we can find that the PSNR of the denoised image improved by not much better than what has been done by the general method [12,13].

(1) Lena Image



(2) Barbara Image

(3) Boat Image



(4) Goldhill Image

Fig. 2. The results of image after denoising with our method (δ=20)

In Fig. 2, original clean images are located in the first column, the second column contains the noisy images and the third one contains denoised images of our proposed algorithm.

5. Conclusion

In this paper we proposed a simple and efficient algorithm for adaptive noise reduction, which combines the linear filtering and thresholding methods in the wavelet transform domain. Experimental results show that our noise reduction algorithm exhibits much better performance in both PSNR and visual effect. The applicability of the proposed algorithm is manifold during the image processing needed, such as display system, HDTV and DMB (Digital Multimedia Broadcasting).

From now on, it is further suggested that the proposed algorithm may be extended to the color images and video framework, which may further improve the video denoising.

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