J Inf Process Syst, Vol.15, No.2, pp.251~260, April 2019
https://doi.org/10.3745/JIPS.01.0039

ISSN 1976-913X (Print)
ISSN 2092-805X (Electronic)

JOURNAL OF INFORMATION PROCESSING SYSTEMS JIPS

Research on Finite-Time Consensus of Multi-Agent Systems

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Abstract

In order to ensure second-order multi-agent systems (MAS) realizing consensus more quickly in a limited time, a new protocol is proposed. In this new protocol, the gradient algorithm of the overall cost function is introduced in the original protocol to enhance the connection between adjacent agents and improve the moving speed of each agent in the MAS. Utilizing Lyapunov stability theory, graph theory and homogeneity theory, sufficient conditions and detailed proof for achieving a finite-time consensus of the MAS are given. Finally, MAS with three following agents and one leading agent is simulated. Moreover, the simulation results indicated that this new protocol could make the system more stable, more robust and convergence faster when compared with other protocols.

Keywords

Finite-Time Consensus, Convergence Speed, Leader-Following, MAS

1. Introduction

Distributed cooperative control of multi-agent systems (MAS) is a fundamental optimization problem and has been used in many fields. There are many industrial applications involve MAS, such as multi-robot cooperation control, unmanned aerial vehicle formation flight control, and underwater autonomy aircraft [1-5]. Therefore, research on MAS distributed cooperative not only has theoretical significance, but also has practical significance. Consensus is one of the most basic problems of the MAS, which has attracted much attention of researchers in many fields [6-8]. The goal of finite-time consensus of dynamic MAS is to design a consensus protocol which can ensure every agent in the MAS achieves a common state in a limited time [9-14].

However, in practical applications, sometimes each agent needs to converge to an expected state, this type of system is called leader-following MAS [15-19]. In this kind of MAS, although leaders behaviors are usually separated from the followers, they have an impact on the followers. Accordingly, the control task of MAS can be achieved by controlling the state of the leader, which is simple and cost efficient. So far, many researchers have paid much attention to tracking problems of the MAS. Sun and Guan [20] designed a finite-time consensus algorithm for MAS under switched and fixed topologies, where they

Manuscript received April 6, 2017; first revision June 5, 2017; accepted August 3, 2017.

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utilized graph theory and homogeneity theory. Ma et al. [21] proposed a novel distributed control law for leader-following time-invariant linear MAS with finite data rate on the basis of distributing priority to every agent in the MAS. Cheng et al. [22] proposed a consensus algorithm for linear leader-following MAS with communication interference as every agent has its own time-varying gain, and this algorithm weakens the influence of the interference.

However, most of the literatures are aimed at finding the conditions to achieve consensus tracking, while ignoring the convergence rate of the MAS. In a control system, the convergence rate of MAS is a very important indicator, which affects the accuracy and real-time of the MAS. In recent years, researchers in various fields have made various attempts to make the agents converge faster in the MAS. Based on the analysis of matrix theory and time frequency domain, Huang et al. [23] proposed two new fast consensus algorithms for the discrete MAS based on local information directed networks. Pan et al. [24] compared the convergence speed of the second-order neighbor algorithm with the general algorithm, and used the second-order neighbor information to investigate the consensus of second-order MAS. In addition, Wang et al. [25] presented a consensus algorithm based on the current and past states, which solved the problem of the fast convergence of the second-order integral dynamic MAS.

Although the methods mentioned above can make the agents in the MAS converge more quickly, it may converge to an agreement in an infinite time. While, systems with high temporal precision are often required to achieve an agreement in a limited time. Finite-time consensus not only has higher precision, but also has the robustness of uncertain factors and strong anti-interference advantages, thus it is a better solution in engineering application. Currently, there are many researches on the topic of finite-time consensus. In [26], the authors proposed two bounded control laws. At the same time, consensus tracking problems for second-order MAS with one and several leaders were investigated. In addition, Lee et al. [27] adopted a fuzzy disturbance observer to study the leader-following problems of heterogeneous MAS.

According to recent findings, there are two methods to enhance the connection between various agents in the MAS, thus improving the convergence rate of agents. One is to change the network topology, and the other one is to acquire more information among other agents. This paper proposes a protocol with the gradient algorithm of the overall cost function which acquires more information and accelerates the convergence rate of the MAS in a limited time.

2. Prior Knowledge and Problem Statement

2.1 Prior Knowledge

Assume that a second-order MAS consists of n agents. Fixed undirected G is represented by $G=(V,E)$, and $V=\{v_1, v_2,\ldots,v_m\}$ represents the network structure of the information exchange through different individuals, in addition, $E=V\times V$. Node v_m represents the m^{th} agent. An edge e_{mn} = $(v_m,v_n)\in E$ in weighted graph G is denoted that the m^{th} agent and the n^{th} agent can sent message to each other directly, the adjacency matrix A=$[a_{ij}]$,(a_{ij}>0,a_{ii}=0). For any $i,j\in V$ in a fixed undirected graph, there is $(v_i=v_j)\in E \Leftrightarrow (v_i=v_j)\in E$, that is to say all real numbers are unordered. The node degree of v_i is $d_i=|N_i|$ and the definition of node degree is $D=diag(d_1,d_2,\ldots,d_m)$. $L=D-A$ represents the Laplacian matrix of weighted graphs. In addition, its spectral property is an important factor to measure the convergence of the protocol. L is a symmetric nonnegative matrix containing a zero value. The rank of L satisfies $rank(L)=n-1$ in an undirected graph G, all of its eigenvalues are non-negative real numbers and all characteristic values are sorted as following:

$$0 = \lambda_1 < \lambda_2 \le \lambda_3 \cdots \le \lambda_m \tag{1}$$

where 0 is eigenvalue of the matrix and $\boldsymbol{1}$ is the corresponding eigenvector ($\boldsymbol{1}=[1,\cdots,1]^T \in \mathbb{R}^n$).

2.2 Problem Statement

Suppose that the second-order MAS contains n agents, and the dynamic model is defined as:

$$\begin{cases} \dot{x}_r(t) = v_r(t), \\ \dot{v}_r(t) = u_r(t), \end{cases} r \in R \tag{2}$$

where $x_r(t) \in R$ represents location information, $v_r(t) \in R$ represents speed information of the agent r, $u_r(t) \in R$ represents control input. The dynamic model of the leading agent in the MAS is defined as:

$$\begin{cases} \dot{x}_o(t) = v_o(t), \\ \dot{v}_o(t) = 0, \end{cases} \tag{3}$$

Sun and Guan [20] studied the second-order MAS with one leader, and proposed a new finite time consensus algorithm:

$$u_i(t) = \sum_{j\in N} a_{ij} sig\left(x_j - x_i\right)^{\delta_1} + \sum_{j\in N} a_{ij} sig\left(v_j - v_i\right)^{\delta_2} + a_{io} sig\left(x_i - x_o\right)^{\delta_1} + a_{io} sig\left(v_o - v_i\right)^{\delta_2} \tag{4}$$

where $i = 1,2,\cdots,n, 0 < a_{ij} < 1$, $0 < \delta_1 < 1$, $\delta_2 = 2\delta_1 / (\delta_1 + 1)$, the value between leader o and follower i is a_{io}, and $sign(y)$ is an expression of the $sign$ function.

This article aims to improve the convergence performance of the MAS, and presents a new protocol with gradient algorithm of the overall cost function for the leader-following second-order MAS:

$$\begin{aligned} u_i(t) &= \sum_{j\in N} a_{ij} sig\left(x_j - x_i\right)^{\delta_1} + a_{io} sig\left(x_o - x_i\right)^{\delta_1} + \sum_{j\in N} a_{ij} sig\left(v_j - v_i\right)^{\delta_2} + a_{io} sig\left(v_o - v_i\right)^{\delta_2} \\ &+ \beta_1 \sum_{j\in N} a_{ij}\left(x_j - x_i\right) + \beta_1 \sum_{j\in N} a_{ij}\left(v_j - v_i\right) + \beta_2 a_{io}\left(x_o - x_i\right) + \beta_2 a_{io}\left(v_o - v_i\right) \end{aligned} \tag{5}$$

where $\beta_1 > 0, \beta_2 > 0$.

DEFINITION 1. The MAS consensus is realized in a limited time if $T_0 \in [0,+\infty)$ is existed, for arbitrary initial state, system (2) satisfies $\lim_{t\to T_o} x_i(t) = x_o(t)$, $\lim_{t\to T_o} v_i(t) = v_o(t)$.

3. Finite Time Consensus Analysis

ASSUMPTION. The undirected connected topology consists of n agents. In addition, there must be a link between leader and followers.

REMARK 1. According to [20], if MAS (2) is asymptotically convergent and the degree of it is same as $\lambda=\delta_1-1<0$ with expansion $(\underbrace{\gamma_i,\dots,\gamma_i}_{n},\underbrace{\gamma_j,\dots,\gamma_j}_{n})$, the system can converge to an agreement in finite time.

LEMMA 1 ([28]). For an undirected graph, if there is a function $\vartheta: R^2 \to R$ satisfies $\vartheta(x_j,x_i)=-\vartheta(x_i,x_j)$, $\forall i,j\in N$, i and j are not equal, there is a group number $z_1,z_2,\cdots,z_n$ satisfying:

$$\sum_{i=1}^{n}\sum_{j\in N} a_{ij} z_j \vartheta(x_j,x_i) = -\frac{1}{2}\sum_{(v_i,v_j)\in E} a_{ij}(z_j-z_i)\vartheta(x_j,x_i)$$

LEMMA 2 ([29]). Suppose that equation $\dot{x}(t)=\zeta(t)$ is solved, $x(t)$ can be obtained. In addition, $\zeta: C\to R^n$ is continuous on an open set C, and C is a subset of R^n. $V: C\to R$ satisfies the condition $D^+V(x(t))\le 0$. Therefore, $\Theta^+(x_o)\cap C$ is part of the whole solutions of $P=\{x\in C: D^+V(x)=0\}$, D^+ indicates upper *Dini* derivative and $\Theta^+ x(0)$ indicates positive limited set.

In general, the homogeneity with expansion used to analysis the finite-time convergence, detailed introduction is given in [30]. n-order MAS has several properties:

$$\dot{x}=\zeta(x), x=(x_1,x_2,\dots,x_n)\in R^n \tag{6}$$

A continuous stream of vectors $\zeta(x)=(\zeta_1(x),\dots,\zeta_n(x))^T$ is the same degree as $\lambda\in R$ with expansion $\gamma=(\gamma_1,\gamma_2,\dots,\gamma_n)$. When $\varepsilon>0$, there will be $\zeta_i(\varepsilon^{r_1}x_1,\varepsilon^{r_2}x_2,\dots,\varepsilon^{r_n}x_n)=\varepsilon^{\kappa+r_i}\zeta_i(x)$.

DEFINITION 2. When the vector flow of MAS (6) is homogeneous, the MAS (6) is homogeneous. In addition,

$$\dot{x}(t)=\zeta(x)+\tilde{\zeta}(x), \tilde{\zeta}(0)=0, x\in R^n \tag{7}$$

$\dot{x}(t)$ is partly the same degree as $\lambda\in R$ with expansion $(\gamma_1,\gamma_2,\dots,\gamma_n)$, if $\zeta(x)$ is the same degree as $\lambda\in R$ with expansion $(\gamma_1,\gamma_2,\dots,\gamma_n)$. Besides, the continuous vector flow $\zeta(x)$ satisfying:

$$\lim_{\varepsilon\to 0}\frac{\tilde{\zeta}_i(\varepsilon^{\gamma_1}x_1,\varepsilon^{\gamma_2}x_2,\dots,\varepsilon^{\gamma_n}x_n)}{\varepsilon^{\kappa+\gamma_i}}=0, \forall x\neq 0, i\in I \tag{8}$$

LEMMA 3 ([31]). Consider that the degree of MAS (2) and $\lambda\in R$ with expansion $(\gamma_1,\gamma_2,\dots,\gamma_n)$ are the same. Function $\zeta(x)$ is continuous, meanwhile, eigenvalue 0 is a gradually stable equilibrium point. At the same time, the MAS (2) is stable in a limited time under the condition that the homogeneous degree $\lambda<0$. In addition, the MAS (7) is partially stable in a limited time if (8) is established.

THEOREM 1. Under Assumption 1, the MAS (2) can achieve consensus tracking in a limited time when consensus tracking algorithm (5) is used.

Proof. Suppose $\overline{x}_i(t)=x_i(t)-x_o(t)$, $\overline{v}_i(t)=v_i(t)-v_o(t)$, then under protocol (5), the system (2) and system (3) become:

$$\begin{aligned}
&\dot{\overline{x}}_i=\overline{v}_i\\
&\dot{\overline{v}}_i=u_i=\sum_{j\in N}a_{ij}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{j\in N}a_{ij}sig(\overline{v}_j-\overline{v}_i)^{\delta_2}-a_{io}sig(\overline{x}_i)^{\delta_1}-a_{io}sig(\overline{v}_i)^{\delta_2}\\
&+\beta_1\sum_{j\in N}a_{ij}(\overline{x}_j-\overline{x}_i)+\beta_1\sum_{j\in N}a_{ij}(\overline{v}_j-\overline{v}_i)-\beta_2 a_{io}(\overline{x}_i+\overline{v}_i)
\end{aligned} \tag{9}$$

Choose five Lyapunov functions $(V_1(t), V_2(t), \ldots, V_5(t))$, and Lyapunov function V can be obtained when these functions are added together.

$$V_1(t)=\frac{1}{2}\sum_{i=1}^{n}\overline{v}_i^2 \qquad V_2(t)=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{\overline{x}_i-\overline{x}_j} a_{ij}sig(s)^{\delta_1}ds \qquad V_3(t)=\sum_{i=1}^{n}\int_0^{\overline{x}_i} a_{io}sig(s)^{\delta_1}ds$$

$$V_4(t)=\frac{\beta_1}{2}\sum_{i=1}^{n}a_{io}\overline{x}_i^2 \qquad V_5(t)=\frac{\beta_2}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)^2$$

Taking into account that the derivative of Lyapunov function along the trace of MAS (9),

$$\dot{V}_1=\sum_{i=1}^{n}\overline{v}_i\dot{\overline{v}}_i=\sum_{i=1}^{n}\overline{v}_i[\sum_{j=1}^{n}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{j=1}^{n}a_{ij}sig(\overline{v}_j-\overline{v}_i)^{\delta_2}$$
$$-a_{io}sig(\overline{x}_i)^{\delta_1}-a_{io}sig(\overline{v}_i)^{\delta_2}+\beta_1\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i+\overline{v}_j-\overline{v}_i)-\beta_2a_{io}(\overline{x}_i+\overline{v}_i)]$$

$$\dot{V}_2=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)sig(\overline{x}_j-\overline{x}_i)^{\delta_1} \qquad \dot{V}_3=\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{x}_i)^{\delta_1},\ \dot{V}_4=\beta_1\sum_{i=1}^{n}a_{io}\overline{x}_i\overline{v}_i$$

$$\dot{V}_5=\frac{\beta_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)(\overline{v}_j-\overline{v}_i)$$

Then,

$$\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3+\dot{V}_4+\dot{V}_5$$
$$=\sum_{i=1}^{n}\overline{v}_i[\sum_{j=1}^{n}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}-\sum_{j=1}^{n}a_{ij}sig(\overline{v}_i-\overline{v}_j)^{\delta_2}-a_{io}sig(\overline{x}_i)^{\delta_1}-a_{io}sig(\overline{v}_i)^{\delta_2}+\beta_1\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)+\beta_1\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)$$
$$-\beta_2a_{io}(\overline{x}_i+\overline{v}_i)]+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{x}_i)^{\delta_1}+\beta_1\sum_{i=1}^{n}a_{io}\overline{x}_i\overline{v}_i+\frac{\beta_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)(\overline{v}_j-\overline{v}_i)$$
$$=\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}sig(\overline{v}_j-\overline{v}_i)^{\delta_2}-\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{x}_i)^{\delta_1}-\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{v}_i)^{\delta_2}+\beta_1\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)$$
$$+\beta_1\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)-\beta_2\sum_{i=1}^{n}a_{io}\overline{v}_i\overline{x}_i-\beta_2\sum_{i=1}^{n}a_{io}\overline{v}_i^2+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{x}_i)^{\delta_1}$$
$$+\beta_1\sum_{i=1}^{n}a_{io}\overline{x}_i\overline{v}_i+\frac{\beta_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)(\overline{v}_j-\overline{v}_i)$$
$$=\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}sig(\overline{v}_j-\overline{v}_i)^{\delta_2}-\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{v}_i)^{\delta_2}+\beta_1\sum_{i=1}^{n}\overline{v}_i\sum_{j=1}^{n}a_{ij}(\overline{v}_j-\overline{v}_i)-\beta_2\sum_{i=1}^{n}a_{io}\overline{v}_i^2$$
$$=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{v}_i-\overline{v}_j)sig(\overline{v}_j-\overline{v}_i)^{\delta_1}-\sum_{i=1}^{n}a_{io}\overline{v}_isig(\overline{v}_i)^{\delta_2}-\frac{\beta_2}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{v}_i-\overline{v}_j)^2-\beta_2\sum_{i=1}^{n}a_{io}\overline{v}_i^2\le 0$$

Note that $\dot{V}(t)=0$ only when condition $\overline{v}_j=\overline{v}_i=0$ is satisfied, thus we get $\dot{\overline{v}}_i=0, \forall i\in I$, as follows:

$$\dot{\overline{v}}_i(t)=u_i(t)$$
$$=\sum_{j\in N}a_{ij}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{j\in N}a_{ij}sig(\overline{v}_j-\overline{v}_i)^{\delta_2}-a_{io}sig(\overline{x}_i)^{\delta_1}-a_{io}sig(\overline{v}_i)^{\delta_2}$$
$$+\beta_1\sum_{j\in N}a_{ij}(\overline{x}_j-\overline{x}_i)+\beta_1\sum_{j\in N}a_{ij}(\overline{v}_j-\overline{v}_i)-\beta_2a_{io}(\overline{x}_i+\overline{v}_i)$$
$$=0$$

Then,

$$\sum_{i=0}^{n}\overline{x}_i[\sum_{j\in N}^{n}a_{ij}sig(\overline{x}_j-\overline{x}_i)^{\delta_1}-a_{io}sig(\overline{x}_i)^{\delta_1}+\beta_1\sum_{j\in N}a_{ij}(\overline{x}_j-\overline{x}_i)-\beta_2a_{io}(\overline{x}_i)]$$
$$=-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)sig(\overline{x}_j-\overline{x}_i)^{\delta_1}-\sum_{i=1}^{n}a_{io}\overline{x}_isig(\overline{x}_i)^{\delta_1}-\frac{\beta_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)^2-\beta_2a_{io}(\overline{x}_i)^2=0 \quad (10)$$

Meanwhile, we can get:

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)sig(\overline{x}_j-\overline{x}_i)^{\delta_1}+\sum_{i=1}^{n}a_{io}\overline{x}_isig(\overline{x}_i)^{\delta_1}+\frac{\beta_1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}(\overline{x}_j-\overline{x}_i)^2+\beta_2a_{io}(\overline{x}_i)\geq 0 \quad (11)$$

In addition, $\overline{x}_j=\overline{x}_i=0$, $\forall i\neq j$ can be obtained from (10) and (11), and from Lemma 3, we can get $x_i-x_o\to 0, v_i-v_o\to 0, \forall i\in I$, $t\to\infty$. By remark 1, we can get MAS (2) and MAS (3) are the same degree as $\lambda=\delta_1-1$ with expansion $(\underbrace{\gamma,\gamma,\ldots,\gamma}_{n},\underbrace{\delta_1+1,\delta_1+1,\ldots,\delta_1+1}_{n})$ when use protocol (5). Therefore, system (2) and system (3) can reach a consensus in a limited time from Lemma 3.

REMARK 2. Obviously, consensus protocol (5) is equal to consensus protocol (4) when $\beta_1=\beta_2=0$. The convergence rate of MAS is faster under consensus protocol (5) when $\beta_1>0, \beta_2>0$.

Then from the following comparison simulations we can see the advantages of consensus protocol (5).

4. Simulations

In this part, we use intuitive simulation to reflect the correctness of the proof and the superiority of the algorithm. Suppose that there are four agents in the MAS, agent 0 represents the leader and others represent the followers, the topology of the weighted undirected graph is shown in Fig. 1.

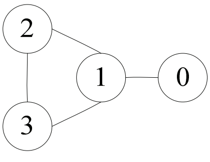
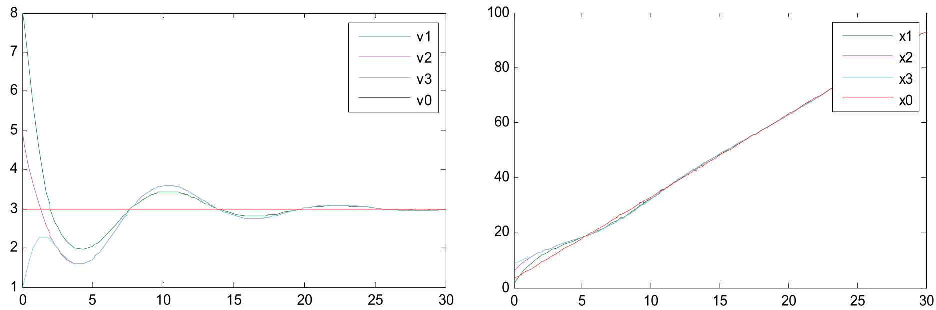


Fig. 1. Network topology.

Assuming that the weight of each edge in Fig. 1 is 1, the location information of the initial state of the leader is $x_o(0)=3$, and the speed information is $v_o(0)=3$. In addition, the location information of the initial state of the three following agents is $x(0)=(1,6,9)^T$, and the speed information is $v(0)=(8,5,1)^T$. Suppose $\delta_1=0.9, \delta_2=2\delta_1/(\delta_1+1)=0.9474$. Four different cases are considered when β_1 and β_2 are of different values, then the fast convergence of the proposed protocol is proved from many aspects.

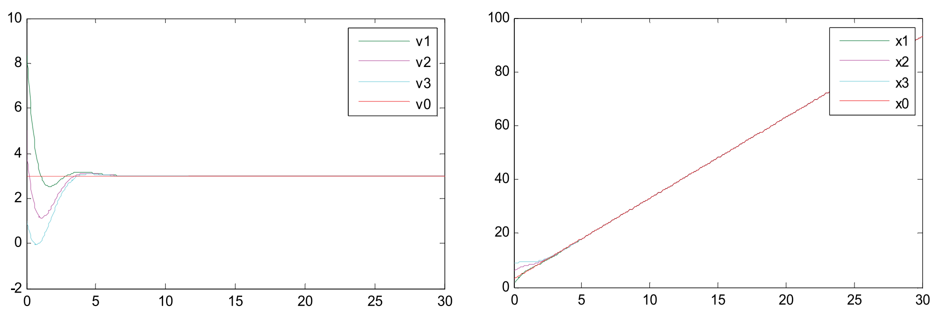
The state of system (2) under protocol (4) is shown in Fig. 2.

When the gradient algorithm of the overall cost function is introduced, the simulation results under protocol (5) are given as follows.



(a) (b)

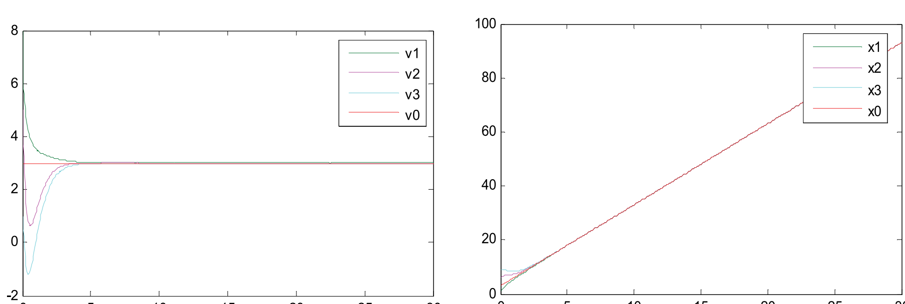
Fig. 2. States of MAS (2) under protocol (4). (a) State information of location. (b) State information of velocity.



(a) (b)

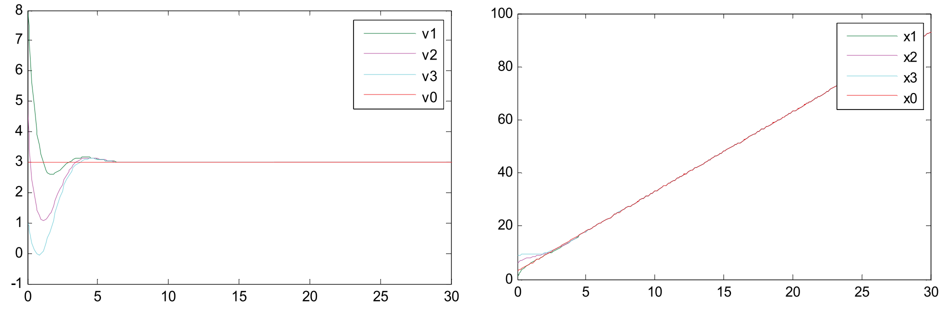
Fig. 3. States of MAS (2) under protocol (5) with $\beta_1 = 15, \beta_2 = 5$. (a) State information of location. (b) State information of velocity.

From Fig. 2, it can be seen that with protocol (4) the multi-agent system took about 28 seconds to achieve an agreement. Taken together Figs. 3–5, it can be seen that the convergence rate improved significantly under the proposed protocol. In Fig. 3, $\beta_1 = 15, \beta_2 = 5$, and the time for MAS to achieve consensus is $T_0 \approx 8s$. In Fig. 4, $\beta_1 = \beta_2 = 5$, and the time for MAS to achieve consensus is $T_0 \approx 5s$. In Fig. 5, $\beta_1 = 5, \beta_2 = 35$, and the time for MAS to achieve consensus is $T_0 \approx 7s$. From Fig. 6, we know that MAS cannot realize consensus in limited time when β_1 and β_2 do not meet the conditions. In summary, we can conclude that when the gradient algorithm of the overall cost function is introduced and only if $\beta_1 > 0, \beta_2 > 0$, the leader-following MAS can achieve finite-time consensus in a faster convergence speed.



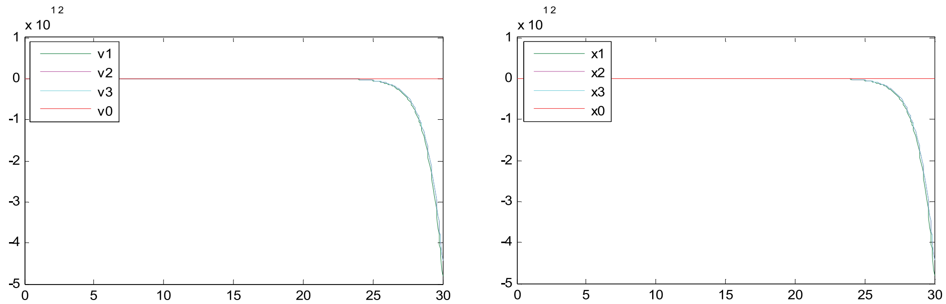
(a) (b)

Fig. 4. States of MAS (2) under protocol (5) with $\beta_1 = \beta_2 = 5$. (a) State information of location. (b) State information of velocity.



(a) (b)

Fig. 5. States of MAS (2) under protocol (5) with $\beta_1 = 5, \beta_2 = 35$. (a) State information of location. (b) State information of velocity.



(a) (b)

Fig. 6. States of MAS (2) under protocol (5) ($\beta_1 < 0$ or $\beta_2 < 0$ or $\beta_1 < 0, \beta_2 < 0$). (a) State information of location. (b) State information of velocity.

5. Conclusion

In this paper, we presented a new finite-time consensus protocol for the second-order MAS. Utilizing Lyapunov theory, graph theory and homogeneity theory, we have verified that the proposed tracking protocol can make MAS realize consensus more quickly in a limited time. The convergence speed of the MAS is improved nearly four times under our conditions. Last, comparison simulation results confirm that the designed protocol makes the MAS reach a finite time consensus with a faster convergence speed, the advantages of the proposed protocol are obvious. The next research direction is to the MAS with noisy interference.

Acknowledgement

The study is supported by National Natural Science Fund (No. 61503073).

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