

Fast Fuzzy Control of Warranty Claims System

Sang-Hyun Lee*, Sung Eui Cho* and Kyung-li Moon**

Abstract—Classical warranty plans require crisp data obtained from strictly controlled reliability tests. However, in a real situation these requirements might not be fulfilled. In an extreme case, the warranty claims data come from users whose reports are expressed in a vague way. Furthermore, there are special situations where several characteristics are used together as criteria for judging the warranty eligibility of a failed product. This paper suggests a fast reasoning model based on fuzzy logic to handle multi-attribute and vague warranty data.

Keywords—Warranty Claims, Age, Usage, Fuzzy Logic

1. INTRODUCTION

Warranty is defined as a contractual obligation of a manufacturer in selling a product to ensure that the product functions properly during the warranty period [2]. Warranty data is a prime source of field reliability data, which is collected economically and efficiently through service networks [3]. It is difficult to capture warranty claims data regarding reliability in polluted and imprecise situations, especially for new and durable products, non-mass products, and short product development times. Usually, there is no comparative reliability information available, the warranty claims data tend to be based on subjective evaluation or rough estimate. To deal with these problems, the modeling of reliability distribution has to be based on the fuzziness of warranty claims data [1, 4]. Many studies also demonstrate that the fuzzy theory is suitable for modeling the reliability property of a product [2]. Therefore, it is more than reasonable to incorporate the fuzzy theory into the warranty claims data distribution for the warranty model [5].

Many factors contribute to product failures that result in warranty claims. The most important factors are the age of the product and the effects of the manufacturing characteristics, time of manufacture and the operating seasons or environments [7]. However, much of the literature on warranty analysis considers failure models which are indexed by a single variable, such as age or usage. Age is measured by calendar time in terms of years, while usage is measured by real operating time in terms of such items as mileage and number of copies. There are special situations where several characteristics are used together as criteria for judging the warranty eligibility of a failed product [8]. In particular, for automobiles, warranty coverage has both age and mileage limits, whichever occurs first (such as a five-year/50,000-mile protection plan). In most cases, the warranty analysis is characterized by a domain in a two-attribute plane with one

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axis representing age and the other axis representing usage- known as a “two-attribute” warranty plan[9]. Now, different tools are required for appropriately modeling the vague and multi-attribute warranty data, and a suitable reasoning methodology is needed to handle these data as well.

In this paper, we suggest another generalization of the classical two-attribute warranty plan. We consider not only fuzzy lifetimes but also situations in which the usage is fuzzy as well. In particular, we improve the fuzzy warranty inference of the two-dimensional warranty claims system. Our reasoning is that special attention should be given to the defuzzification process when available computational capabilities are restricted by equipment size or cost in the fuzzy warranty inference system. Defuzzification, such as the “center of gravity” determination [11], can be used to arrive back at a “crisp” cost structure, which may be more tangible and easier to work with. The center of gravity, which is probably the best known defuzzification method, provides a single value by calculating the center of gravity for the area under the membership function. This approach is a good solution, but presents two limitations. First, the output area has to be decomposed only in triangular and trapezoidal types, which may not be possible when the membership functions do not have a triangular form. Second, this technique needs to spend computational time determining if each shape corresponds to a triangle or trapezoid.

2. CLASSICAL WARRANTY IDENTIFICATION

In this section, we discuss the classical method to estimate the number of claims for a specific warranty at various combinations of time/mileage limits of the warranty. Although the method focuses on analysis at the component/warranty system level, the estimates at system level can be easily obtained by the pooling of component/warranty-system level estimates.

2.1 Usage accumulation & Repeat Claims

The decisions and actions during design, manufacturing, and assembly determine the inherent reliability of the product [5]. Two-attribute warranty coverage is generally stated in terms of mileage and time limits. It expires when any of the two limits is crossed. Warranty limit denoted as M/K would henceforth refer to the base warranty coverage meaning M months or K usage, whichever occurs first. The warranty cost for any coverage depends on the number of claims and cost of repairing each warranty claim. A main objective of the estimation method being discussed is to be able to assess the impact of changes in time and/or mileage limits on the number of claims. Choosing a dataset known to have an unusually high number of special causes due to design or manufacturing issues and resulting in inflated figures of warranty claims should be avoided.

Such a dataset will give rise to highly unstable estimates with poor statistical properties. A reasonable choice must be a recent product where all products have completed the base warranty period and a product that does not have a very high number of special cause related issues. Let M_i denote time (for instance, in months from the sale of the automobile) and K_i mileage for the i -th ($i=1, \dots, N$) automobile in a population of the same type of vehicles. Let α_i (in miles per month) denote mileage accumulation of i^{th} product. Let $G(\alpha)$ and $g(\alpha)$ be its distribution function (df) and probability density function (pdf), respectively. Estimating $g(\alpha)$ and $G(\alpha)$ is a crucial step in the analysis. A warranty database usually contains mileage accumulation data for

only those products that fail within the warranty period.

Repeat claims could be the result of either a new failure or difficulty in root cause elimination during the previous repair. The expected number of total claims can be obtained by combining estimates of repeat claims with the estimates for the first claims. Repeat claims as a proportion of the first claims can be estimated using the following formula.

$$P_{rc,m/k_0} = (n(m) - n_f(m)) / n_f(m) \quad (1)$$

Here, P denotes the estimate of repeat claims as a proportion of the first claims at month-in-service and K_0 mileage warranty limit, $n(m)$ the total number of warranty claims up to m month-in-service value, and $n_f(m)$ the number of first claims up to m month-in-service. In the absence of any data on repeat claims beyond the existing base warranty period, it is assumed that the repeat claims would continue to grow at a rate established by the fitted polynomial. $C/100$ is analogous to repair per 1000 ($R/100$) terminology used in some industries. However, $R/100$ does not capture all the multiple repairs; in reality, it captures only the number of claims made per 100 products sold. Thus the term repairs/100 is a misnomer and claims/100 is a more appropriate term to use.

2.2 Warranty Cost

The cost per product sold mainly includes time taken by the service technician to successfully repair the product and the material cost. The absolute value of cost per repair may also depend on the nature of failure mode. For example, the cost of repairing an oil level indicator assembly may not exceed \$20 to \$30. On the other hand, the cost to repair a vehicle experiencing an engine failure may run into several thousand dollars. Obtaining warranty cost per unit sold for a warranty system requires estimates of expected $C/100$ and the cost per repair at different month-in-service values. In this section, a method for arriving at warranty cost per unit sold using estimates for $C/100$ and incremental cost per repair is discussed. Repair $C_r(m)$ at any month-in-service m ($m=1, \dots, M$) obtained from the base warranty period M/K is given as equation (2). Here $C(m)$ denotes total warranty cost up to m , $n(m)$ total warranty claims up to m , and $C(0) = n(0) = 0$.

$$C_r(m) = ((C(m) - C(m-1)) / (n(m) - n(m-1))) \quad (2)$$

A curve fitted to $C_r(m)$ versus t captures the changes in warranty cost per repair as a function of month-in-service. A curve fitted to $C_r(m)$ versus m captures the changes in warranty cost per repair as a function of month-in-service. There are numerous factors that impact cost of repairs. Some of the major factors are nature and time of the occurrence of a failure mode, mileage accumulation rates, and the design actions such as design for serviceability. The parametric model fitted to the data gives information regarding nature of the failure mode under study.

3. FUZZY WARRANTY SYSTEM DESIGN

Based on the warranty analysis of many articles, it is concluded that the warranty analysis is a matter of fuzzy control. Fuzzy inference systems have been widely applied to process control. In

several applications, the dynamics of the system are fast enough to make difficult a good control performance when using typical computational equipment. Fig. 1 represents the overall procedure for multi-attribute warranty control, which consists of four components, namely, a fuzzy rule base, a fuzzy inference process, a fuzzification process, and a defuzzification process. The basic unit of any fuzzy system is the fuzzy rule base. All other components of the fuzzy logic system are used to implement these rules in a reasonable and efficient manner. The fuzzy inference process combines the rules in the fuzzy rule base and then carries out a mapping from fuzzy set A in the universe of discourse U to fuzzy set B in the universe of discourse V . Owing to the fact that in most applications the input and output of the fuzzy system are real-valued numbers, one must construct interfaces between the fuzzy inference process and the environment. These interfaces are the fuzzification and defuzzification processes.

The fuzzification process can be defined as a process of mapping a real-valued point $x \in U \subset R^n$ to a fuzzy set A in U . The defuzzification process can be defined as a process of mapping from fuzzy set B in $V \subset R$ (which is the output of the fuzzy inference process) to a crisp value $y \in V$. Fuzzy rules are usually formulated as the following.

IF (Antecedent₁) *OP* (Antecedent₂) ... *OP* (Antecedent_n) THEN (Consequent) (w)

Here, n is an integer, *OP* stands for operators like AND, OR, etc., and w represents a weight value indicating the importance of a rule.

Our fuzzy inference process is based on the following two assumptions: first, all activations of an input fuzzy set are regarded to be a piece of (fuzzy) evidence supporting the domain knowledge an expert formulated via rules and fuzzy sets. Second, each piece of evidence should be incorporated more actively in the decision-making process. These assumptions can be implemented in 3 steps, relating to accumulation, normalization, and decision-making (see [4]). For example, an application of these 3 steps on the example illustrated in Fig. 2 leads to Table 1; the accumulation of the pieces of evidence produces: $Cost_{low} = 0.86$, and $Cost_{normal} = 0.37 + 0.77 = 1.14$. Normalization of these values generates: $Cost_{low} = 0.75$ and $Cost_{normal} = 1.00$. The method therefore produces the outcome: $Cost = normal$. This approach can also be applied to aggregation of the consequents across the rules, as there are many different weights indicating the importance of the rule.

In the fuzzy warranty inference process, special attention should be given to the defuzzification process when available computational capabilities are restricted by equipment size or cost. In

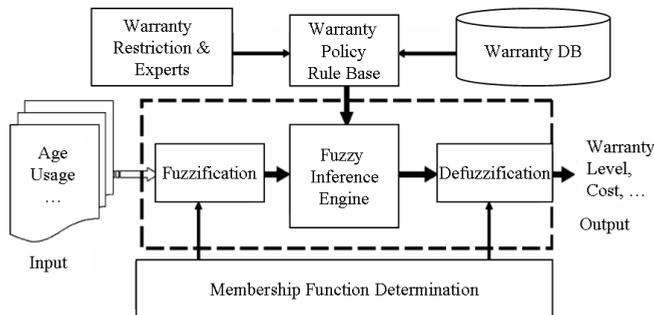


Fig. 1. fuzzy warranty control system

Table 1. Example of 3 steps

	Warranty Cost		
	low	normal	High
Age	0.86	0.37	
Usage		0.77	
Accumulation	0.86	1.14	
Normalization	0.75	1.00	
Decision making	Warranty Cost = Costnormal/1.00		

these cases, the computational time must be reduced in order to improve the warranty controller performance. Hence, it is important to use fast defuzzification methods. One of the most accepted defuzzification methods, when using Mamdani fuzzy inference systems, is based on computing the centroid of the output area. However, the relatively high computational requirements could be a limitation for several applications. As an alternative, faster and simple methods can be used, such as finding the mean of maxima or by finding the half-area point. In connection with the defuzzification method, we present a simple fast method for computing a centroid approximation by fitting the fuzzy warranty output area into a triangular type. The centroid position of any triangular type could be computed as $y_c=(a+b+y_{max})/3$ in figure 2(1) (see [11]). Here, $[a, b]$ denotes a fuzzy warranty output area, and y_{max} a centroid approximation. This approach is a good solution, but presents two limitations. First, the output area has to be decomposed only in triangular and trapezoidal types, which may not be possible when the membership functions do not have a triangular form. Second, this technique needs to spend computational time determining if each shape corresponds to a triangle or to a trapezoid.

In this study, a heuristic approach is used, which can even be applied for a wider set of membership functions in view of the warranty set. This work consists of adapting any output shape into one single triangle. For the triangular approximation, y_c can be rewritten as $(b_R-b_L)/3+ y_{max}$, where b_R is a distance from y_{max} to b , and b_L is a distance from a to y_{max} . When the fuzzy output presents more than one maximum, the location of the triangle maximum is computed as the average of maxima as shown in figure 2(2). First, find the first non-zero value, the last nonzero value and the maxima. This can be done in one single computation loop. Next, find the average in the case of several maxima. Finally, determine b_R and b_L . The computational time required by this approximation is reduced with respect to that of any other methods. For fuzzy outputs not located at the origin, the triangular shape maximum position is located at the maximum output

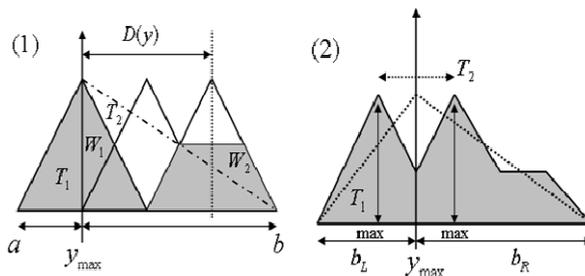


Fig. 2. Output fitting to triangular types

shape position. When the fuzzy output presents more than one maximum, the location of the triangle maximum is computed as the average of maxima.

4. APPLICATION

For the results of this we have implemented our method in MATLAB 7.0, and our tested approach on Tables 2 and 3 summarize the number and amount of warranty claims issued against vehicle A-type shipped out in the month of January 2005. In general, most warranty claims are issued between the ages of 1 to 3 which corresponds to a mileage of 15,000 to 35,000 km. If you consider the covariate called the warranty cost, there is no linear relationship between the age of a vehicle and its mileage. In other words, older age does not mean its mileage is higher. In Korea, the cost for warranty claims varying with mileage is higher than with age.

Figure 3 illustrates the fuzzy types for the input variables “Age” and “Mileage”. The fuzzy type of age is categorized into A1 (low, less than 12 months), A2 (normal, between 12 and 24 months), A3 (more or less high, between 24 and 36 months), and A4 (high, 36 months or older) while the fuzzy type of mileage is divided into M1 (low, 20,000 km or less), M2 (normal, between 20,000 and 40,000 km), M3 (more or less high, between 40,000 and 60,000 km), and M4 (high, 60,000 km or higher).

Fig. 4 represents the fuzzy type in terms of the output variable, i.e. the warranty cost. According to Table 3, the entire warranty cost amounts to \$749,469 while the mean of each row or column is \$187,367.25. Therefore, the fuzzy group in terms of the output variable is composed of C1 (low, \$187,367.25 or less), C2 (normal, between \$187,367.25 and \$374,734.5), C3 (more or less high, between \$374,734.5 and \$562101.75, and C4 (high, \$749,469 or higher). The fuzzy value corresponding to each cell in Table 3 can be obtained by multiplying the cell value by four. Table 4 represents the rule sets formed by the if-then rule.

Table 2. Two way table of warranty counts

Division		Mileage (km)			
		0~ 20000	20000~ 40000	40000~ 60000	60000~ 80000
Age (month)	~ 12	5,179	484	1	0
	12~24	2,533	3,643	91	1
	24~36	794	2,377	1,169	308
	36~60	142	647	740	366

Table 3. Two way table of warranty counts

Division		Mileage (km)			
		0~ 20000	20000~ 40000	40000~ 60000	60000~ 80000
Age (month)	~ 12	238,484	21,973	81	0
	12~24	128,706	179,547	8,977	5
	24~36	26,773	70,402	32,932	5,031
	36~60	2,777	10,638	15,892	7,251

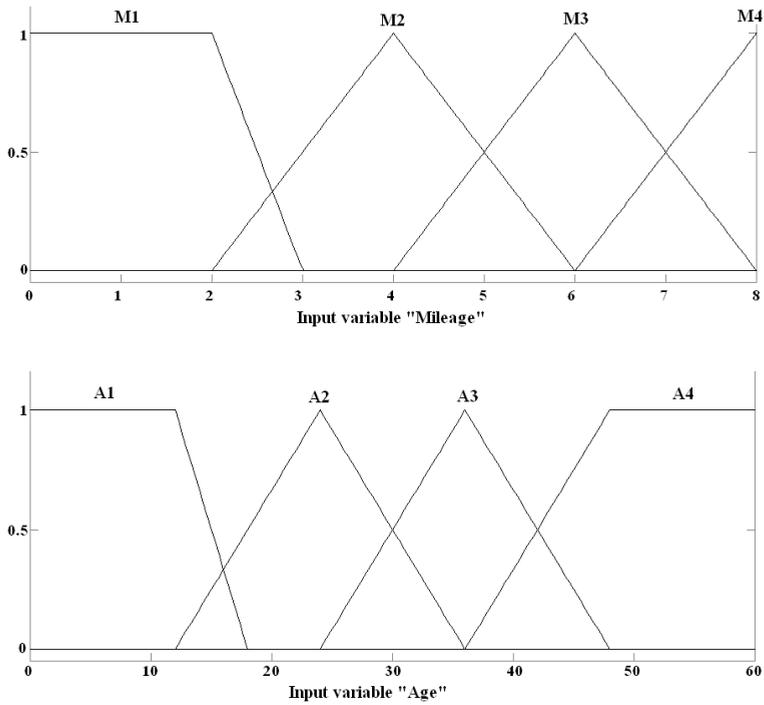


Fig. 3. Output fitting to triangular types

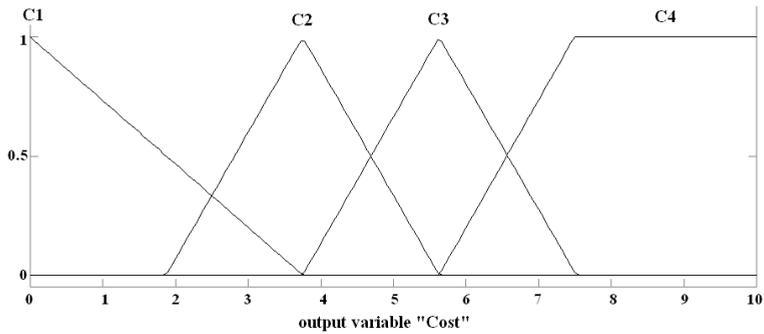


Fig. 4. Fuzzy types for output variables

Table 4. Two way table of warranty counts

Fuzzy Rule		Mileage			
		M1	M2	M3	M4
Age	A1	C4	C1	C1	C1
	A2	C3	C4	C1	C1
	A3	C1	C2	C1	C1
	A4	C1	C1	C1	C1

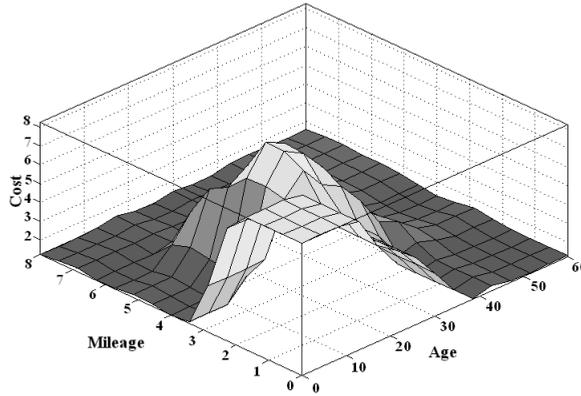


Fig. 5. Surface based on 16 rules

According to Table 2, the number of warranty claims corresponding to the fuzzy value of the input variable of age is 5,664, 6,268, 4,648, and 1,985 and their relative ratios are 0.31, 0.34, 0.25, and 0.10. If you normalize this data with respect to A2, then you may have the weighted ratios of 0.90, 1.0, 0.74, and 0.32. Similarly, if you normalize the input variable, mileage with respect to M1, then you may have the weighted ratios of 1.0, 0.83, 0.24, and 0.08. The following is the fuzzy rule for deduction of the cost for warranty claims.

Fig. 5 represents the fuzzy deduction surface to which sixteen rules and normalized weighted ratios are applied. The deduction surface can be used as a type of continuous approximation distribution for Table 3. The results for deduction are as follows: If the mileage lies between 60,000 and 80,000 km, the result for fuzzy deduction is almost in accordance with the values shown in Table 3. If the age is between 24 and 36 months while the mileage is below 60,000 km, then the cost for warranty claims is relatively low. If the age is between 12 and 24 months while the mileage is below 60,000 km, then the cost for warranty claims is relatively high

5. CONCLUSION

In this paper, we suggested a fast warranty control system based on fuzzy logic. The suggested system is applied to a particular automotive warranty system in Korea. Using the conventional warranty analysis method for the estimation of the warranty cost, the result is relatively higher. The main reasons are attributed to some inaccurate warranty data and special situations where several characteristics are used together as criteria for judging the warranty eligibility of a failed product. The proposed fuzzy warranty system showed more reasonable results in the two-attribute automotive warranty application. For automobiles, the “two-attribute” warranty policy is very important because sometimes warranty coverage has both age and mileage limits. For instance, the rate of an increase in the number of claims changes more rapidly while the accumulated ratio of usage is higher. Finally, since there are various characteristics used together affecting the warranty system, there is room for future improvements on this study. It is critical to probe deeply into the formation of the fuzzy set types in terms of input and output variables. In addition, it is desirable to obtain more accurate measurements for the weighted ratios of the fuzzy rules.

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